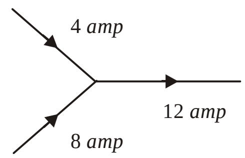
On the basis of magnitude, direction and rules of addition, all physical quantities are classified in to two groups as : Scalars and Vectors:

Scalars

A scalar quantity is one whose specification is completed with its magnitude only. Two or more than two similar scalar quantities can be added according to the ordinary rules of algebra.

For Example : mass, distance, speed, energy, electric flux, current electricity etc.

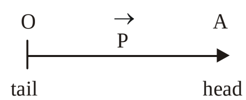
In the above example, current electricity has magnitude and direction both but it is a scalar quantity because two different electrical currents can be added only with simple algebra, as



Vectors

A vector quantity is one whose specification is completed with its magnitude and direction both. Two similar vector quantities can be added according to the law of paralellogram or triangle law. For example displacement, Velocity, acceleration, force, electric field intensity, current density etc.

A vector quantity can be represented by a line. The front end (arrow head) represents the direction and length of the line gives its magnitude as :



; $\vec{P} = \overrightarrow{OA}$

OA = magnitude of the vector (not according to scalar). The magnitude of a vector $\vec{P}$ can be written as $|\vec{P}|$, modulus of $\vec{P}$.

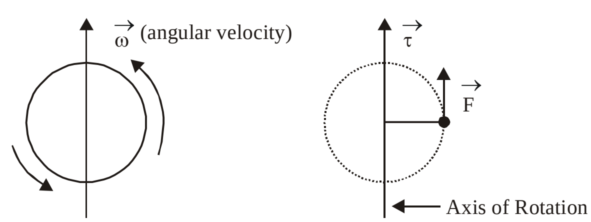
Type of Vectors

(a) Polar Vectors : A vector whose direction is along the direction of the motion of a body or particle is known as a polar vector.

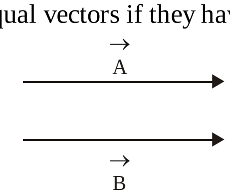
For Example : Displacement, velocity, linear momentum, force etc. are Polar Vectors.

(b) Axial Vecotrs : A vector whose direction is along the axis of rotation of the body or a particle is called an axial vector. An axial vector always produces rotational effect on the body.

For Example : Angular velocity $(\vec{\omega})$, angular accleration $(\vec{\alpha})$, torque $(\vec{\tau})$, angular momentum $(\vec{L})$ are axial vectors.

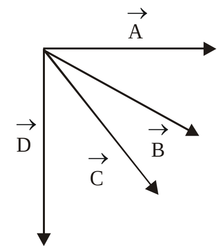


(c) Equal vectors : Two vecors are said to be equal vectors if they have equal magnitude and same directions.



The vector $\vec{A}$ and $\vec{B}$ are equal vectors. i.e., $\vec{A} = \vec{B}$.

(d) Co-initial vectors : Vectors having common initial point are called co-initial vectors. The vectors A, B, C and D are said to b co-initial vectors.



(e) Unit Vectors : A vector of unit magnitude and whose direction is the same as that of the given vector is called unit vector. Basically unit vecotr represents the direction of the given consider a vector $\vec{A}$. This vector can be written as :

vector = (Magnitude of the vector) $\times$ (direction of vector).

or $\vec{A} = |\vec{A}|\ \vec{A}$.

$\hat{A}$ is the unit vector drawn in the direction of $\vec{A}$.

$$\therefore \quad \vec{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\text{Vector}}{\text{Magnitude of the vector}}$$

(f) The zero vector or Null vector : A vector whose magnitude is zero and no sense of direction is called zero vector or null vector. It represented by $\vec{O}$.

For Example : The position vector of origin, the acceleration of a particle moving with uniform velocity etc.

(i) Addition or subtraction of zero vector from a given vector does not affect the given vector.

i.e., $\vec{A} + \vec{O} = \vec{A}$

and $\vec{A} - \vec{O} = \vec{A}$.

If a zero vector multiplied by a scalar number gives the zero vector i.e. $n\ \vec{O} = \vec{O}$.

Multiplication of a Vector by a Real Number

The multiplication of a vector by a scalar quantity *n* gives a new vector whose magnitude is *n* times the magnitude of the given vector. Its direction is same as that of given vector if *n* is a positive real number. Suppose a vector $\vec{a}$ is multiplied by scalar quantity *n*.

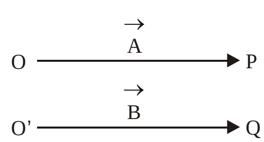
$\therefore \quad \vec{A} = n\ \vec{a}$.

For example : if $n = 4$ than $\vec{A} = 4\vec{a}$.

If $n = -4$ then $\vec{A} = 4\vec{a}$. If $n = 0$ then $\vec{A} = 0$ (null-vector)

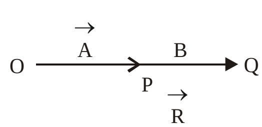
Additional of Vectors

(i) Addition of Collinear vector : suppose $\vec{A}$ and $\vec{B}$ are two collinear vectors.



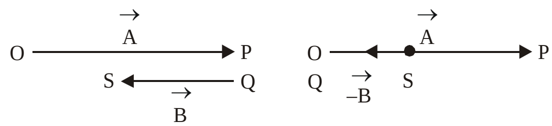
Now the resultant vector $\vec{R} = \vec{A} + \vec{B}$.

i.e.,

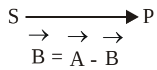


For Example : Suppose a body is displaced through 4 *m* due west and then it is further displaced through 6 *m* due west. Then the resultant displacement of the body = (4 *m* + 6 *m*). = 10 *m* due west.

(ii) Addition of two anti collinear Vectors : Suppose $\vec{A}$ and $\vec{B}$ are two anti colliear vectors as :



i.e.,

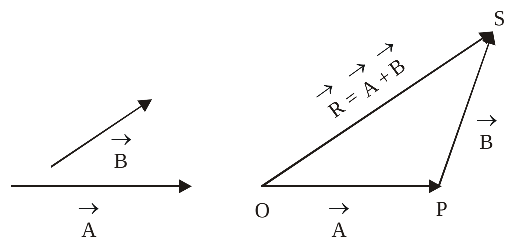


For Example : Suppose a body is displaced through 4 *m* due east and then 2 *m* due west

$\therefore$ Resultant displacement vector $\vec{R}$ = (4 *m* – 2 *m*) = 2 *m* due east.

(iii) Addition of two vectors pointing in different directions: When two vectors are pointing in different directions, they can be added using laws of vector addition as triangle law, parallelogram law and polygon law.

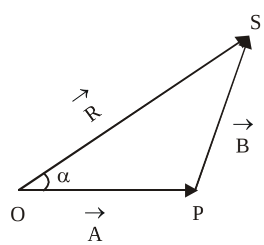
For Example : Suppose $\vec{A}$ and $\vec{B}$ are two vectors



hence $\overline{OS} = \overline{OP} + \overline{PS}$ or $\vec{R} = \vec{A} + \vec{B}$

Triangle law of vector Addition

If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these vectors is represented both in magnitude and direction by the third side of the triangle as shown below.

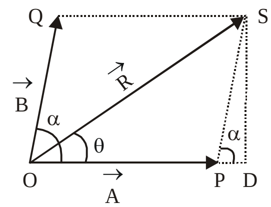


$\overline{OS} = \overline{OP} + \overline{PS}$ or $\vec{R} = \vec{A} + \vec{B}$

The resultant $\vec{R}$ can be calculated by the law of parallelogram.

Parallelogram Law of Vector Addition

The law of parallelogram for vector addition is applied when the two vectors act on the same point simultaneously at a certain angle.



Suppose OPSQ be a parallelogram in which vectors $\vec{A}$ and $\vec{B}$ are represented by the two adjacent sides OP and OQ of the parallelogram. The diagonal $\overrightarrow{OS} = \vec{R}$ represents the resultant of $\vec{A}$ and $\vec{B}$. PD is dropped perpendicular to OP produced.

In the given figure

$$OS^2 = OD^2 + DS^2 \text{ (Pythagorus theorem)}$$

$$OS^2 = (OP + PD)^2 + DS^2 \qquad \text{--- (i)}$$

Now OS = R, magnitude of $\vec{R}$.

OP = A, magnitude of $\vec{A}$.

PS = B, magnitude of $\vec{B}$.

From right angled triangle PDS.

$$\cos\alpha = \frac{PD}{PS}$$

or $PD = PS.\cos\alpha = B.\cos\alpha$ --- (ii)

and $\sin\alpha = \frac{DS}{PS}$

$DS = PS.\sin\alpha = B.\sin\alpha$ --- (iii)

$\therefore$ Equation (i) becomes

$$R^2 = (A + B\cos\alpha)^2 + (B\sin\alpha)^2$$

$$R^2 = A^2 + B^2\cos^2\alpha + 2AB\cos\alpha + B^2\sin^2\alpha$$

$$= A^2 + B^2(\sin^2\alpha + \cos^2\alpha) + 2AB\cos\alpha$$

$$\therefore \quad R^2 = A^2 + B^2 + 2AB\cos\alpha$$

or $R = \left[A^2 + B^2 + 2AB\cos\alpha\right]^{\frac{1}{2}}$

Let $\vec{R}$ make an angle θ with the direction of $\vec{A}$.

From the right angles ΔODS

$$\tan\theta = \frac{DS}{OD} = \frac{DS}{OP + PD} = \frac{B\sin\alpha}{A + B\,\text{con}\,\alpha}$$

$$\theta = \tan^{-1}\left[\frac{B\sin\alpha}{A + B\cos\alpha}\right]$$

This gives the direction of $\vec{R}$.

Special Cases

1. If $\vec{A}$ and $\vec{B}$ act in the same direction,

then $\alpha = 0°$; then R = A + B

and $\tan\theta = \dfrac{B\sin\alpha}{A + B\cos\alpha} = \dfrac{0}{A+B} = 0$

Hence the resultant vector points in the direction of the given vectors.

2. When $\alpha = 180°$, $\vec{A}$ and $\vec{B}$ are antiparallel.

$\therefore$ $R = A - B$, or $R = B - A$

also $\tan\theta = \dfrac{B\sin\alpha}{A + B\cos\alpha} = \dfrac{B\sin 180°}{A + B\cos 180°}$

$= \dfrac{0}{A-B} = 0$

or $\theta = 0°$ or $180°$

when $A > B$ then $\theta = 0°$;

when $B > A$, $\theta = 180°$

i.e., the direction of resultant vector is opposite to the direction of the vector whose magnitude is smaller.

3. When $\alpha = 90°$

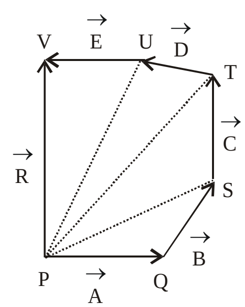
$\therefore$ $R = \sqrt{A^2 + B^2}$

also $\tan\theta = \dfrac{B}{A}$

$\therefore$ $\theta = \tan^{-1}\left(\dfrac{B}{A}\right)$

Polygon law of Vector Addition

If a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, then the redultant vector is represented both in magnitude and direction by the closing side of the polygon taken in the opposite order.



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$

Properties of Vector Addition

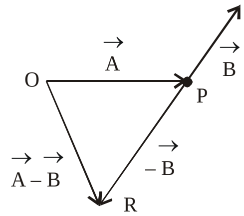
(i) Vector addition is commutative : $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(ii) Vector addition is associative : $\left(\vec{A} + \vec{B}\right) + \vec{C} = \vec{A} + \left(\vec{B} + \vec{C}\right)$

(iii) Vector addition is distributive : $\lambda\left(\vec{A} + \vec{B}\right) = \lambda\vec{A} + \lambda\vec{B}$

Subtraction of Vectors :

The subtraction of a vector $\vec{B}$ from the vector $\vec{A}$ is same as adding $\vec{B}$ in $\vec{A}$.

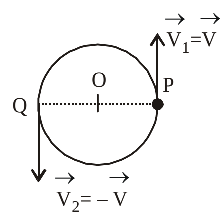


According to triangle law of vector addition, we have from ΔOPR

$$\overline{OR} = \overline{OP} + \overline{PR} = \vec{A} + \left(-\vec{B}\right) = \vec{A} - \vec{B}$$

Illsustration on subtraction of vectors

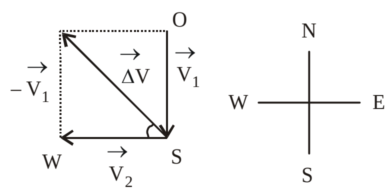
(i) A partical is moving with constant speed in a circular orbit. Find the change in its velocity when it completes half the revolution.



when a particle moves along a circular path with a constant speed then its velocity changes due ot change in direction.

$\therefore$ change in velocity, $\Delta\vec{V} = \vec{V}_1 - \vec{V}_2 = \vec{V} - \left(-\vec{V}\right) = 2\vec{V}$

(ii) A car moving towards the south changes its direction towards the west direction with the same speed, Find the change in the velocity of the car.



Here $\left|V_1\right| = \left|V_2\right| = v$ (say)

$\therefore$ Change in velocity of car, $\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$

Magnitude of the change in velocity

$$\left|\Delta\vec{V}\right| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos 90°} = \sqrt{v^2 + v^2 + 0}$$

$$= \sqrt{2v^2} = \sqrt{2}v$$

The direction of change in velocity,

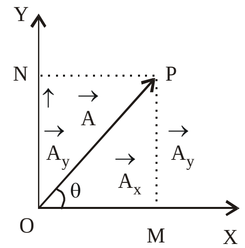
$$\tan\theta = \frac{\left|\vec{V}_1\right|}{\left|\vec{V}_2\right|} = \frac{v}{v} = 1 \text{ or } \theta = 45°$$

the change in velocity of th car is along north-west direction.

Resolution of Vectors

A. In two perpendicular components

When a vecotr is resolved into its components and the components are at right angles to each other then such components are called rectangular components.



Suppose $\vec{A}_x$, and $\vec{A}_y$ are rectangular components of $\vec{A}$. According to triangle law of vector addition,

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \overrightarrow{OM} + \overrightarrow{ON}$$

$$\therefore \quad \vec{A} = \vec{A}_x + \vec{A}_y$$

Here $\vec{A}_x$ and $\vec{A}_y$ are two rectangular components of A. If $\hat{i}$ and $\hat{j}$ be the unit vectors along *x*-axis and *y*-axis respectively, then

$$\vec{A} = \vec{A}_x \hat{i} + \vec{A}_y \hat{j}.$$

If θ is the angle subtended by Vector $\vec{A}$ with *x*-axis, $A_x = A\cos\theta$, and $A_y = A\sin\theta$, represented the rectangular components of A along two perpendicular directions.

$$\therefore \quad A^2 = A_x^2 + A_y^2$$

$$\text{or} \quad A = \sqrt{A_x^2 + A_y^2}$$

$$\therefore \quad \tan\theta = \frac{A_y}{A_x}$$

$$\text{or} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Example : A force of 8N makes an angle 30° with *x*-axis. Find the *x* and *y* components of the force.

Solution : Here F = 8N, $\theta = 30°$

x-component of force, $F_x = F\cos\theta = 8\times\frac{\sqrt{3}}{2} = 4\sqrt{3}$ N

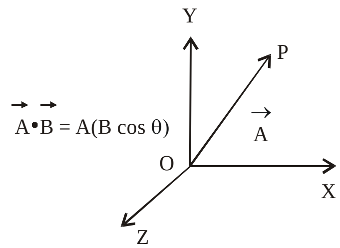
y-component of force, $F_y = F\sin\theta = 8\times\frac{1}{2} = 4$N

Resolution of a Vector into three Rectangular Components

Suppose a vector $\vec{A}$ in space as shown in the figure. Let the rectangular component of $\vec{A}$ along *x*-axis, *y*-axis and *z*-axis are $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ respectively.

According to the vector addition rule it may be written as

$$\vec{A}\cdot\vec{B} = A\,(B\cos\theta)$$



$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along *x*, *y* and *z* axis respectivley,

then $\vec{A}_x = A_x\hat{i}$, $\vec{A}_y = \vec{A}_y\,\hat{j}$ and $\vec{A}_z = A_z\,\hat{k}$.

$$\therefore \quad \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\therefore \quad |\vec{A}|^2 = A^2{}_x + A^2{}_y + A^2{}_z ,$$

$$\therefore \quad |A| = \sqrt{A_x{}^2 = A_y{}^2 + A_x{}^2}$$

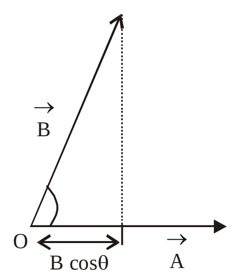
PRODUCT OF VECTORS

Two vectors can be multiplied in the following two ways.

Scalar Product or Dot Product

The scalar product of two vector $\vec{A}$ and $\vec{B}$ is defined as the product of magnitude of $\vec{A}$ and $\vec{B}$ multiplied by the cosine of the smaller angle between them.

Then $\vec{A}\cdot\vec{B} = A\,B\cos\theta$



or $\vec{A}\cdot\vec{B} = A\,(B\cos\theta)$ = (magitute of $\vec{A}$) (component of $\vec{B}$ in the direction of $\vec{A}$).

Dot product or scalar product of two vectors gives the scalar quantity.

Example:

(i) The dot product of force $(\vec{F})$ and displacement $(\vec{S})$ gives work. (scalar quantity) $\vec{F}\cdot\vec{S} = W$

(ii) The dot product of force $(\vec{F})$ and velocity $(\vec{V})$ is equal to power (scalar quantity) i.e. $\vec{F}\cdot\vec{V} = P$.

(iii) The dot product of magnetic induction $(\vec{B})$ and area vector $(\vec{A})$ is equal to the magnetic flux (φ) linked with the surface (scalar quantity) $\vec{B}\cdot\vec{A} = \varphi_B$.

(iv) The dot product of electirc field intensity $(\vec{E})$ and area vector $(\vec{A})$ is equal to the electric flux (ϕ_E) linked with the surface (scalar quantity) $\vec{E}\cdot\vec{A} = \phi_E$.

Popertise of Scalar Product of Dot Product

(i) Dot product is commutative :

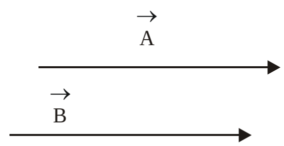
$\vec{A}\cdot\vec{B}$ = AB cos θ and $\vec{B}\cdot\vec{A}$ = BA cos θ = AB cos θ

$\therefore$ $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ which is commutative law.

(ii) Dot product is distributive over the addition of vectors :

i.e., $\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$

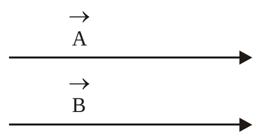
(iii) Dot product of two parallel vectors :



Here θ = 0°

$\therefore$ A · B = AB cos 0° = AB $\qquad [\because \cos 0° = 1]$

(iv) Dot product of two equal vectors :



the angle between two equal vectors is zero.

i.e., θ = 0°

$\therefore$ cos 0° = 1

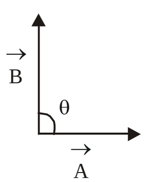
$\therefore$ $\vec{A}\cdot\vec{A}$ = A A cos 0° = A^2

Similarly $\hat{i}\cdot\hat{i} = 1\times1\times\cos 0° = 1$, $\hat{j}\cdot\hat{j} = 1\times1\times\cos 0° = 1$

$\hat{k}\cdot\hat{k} = 1\times1\times\cos 0° = 1$

$\therefore$ $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

(v) Dot product of perpendicular vectors :



θ = 90°

$\therefore$ cos 90° = 0

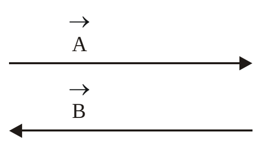
If two vectors $\vec{A}$ and $\vec{B}$ are perpendicular then $\vec{A}\cdot\vec{B} = \vec{A}\vec{B}$ = AB cos 90° = 0

Now $\hat{i}\cdot\hat{j} = 1\times1\times\cos 90°$, $\hat{j}\cdot\hat{k} = 1\times1\times\cos 90°$, $\hat{k}\cdot\hat{i}$

$= 1\times1\times\cos 90° = 0$

Thus $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0°$

(vi) Dot product of two antiparallel vectors:



Here θ = 180°'

$\therefore$ cos 180° = – 1

Then $\vec{A}\cdot\vec{B}$ = AB cos 180° = – AB

(vii) Dot product of two vectors in terms of their components:

If $\vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{B} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\therefore \quad \vec{A}\cdot\vec{B} = \left(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}\right)\cdot\left(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}\right)$

or $\quad \vec{A}\cdot\vec{B} = x_1x_2 + y_1y_2 + z_1z_2$

where $\quad \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

and $\quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Vector Product or cross Product

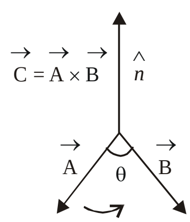
The vector product of two vectors is a vector quantity. The cross product of two vectors is a single vector whose magnitude is equal to the product of the magnitudes of two given vectors multiplied by the sine of the smaller angle between two given vectors.

The direction of the vector given by the cross product of the two vectors is perpendicular to the plane containing the two vectors.

i.e., $\vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{n} = \vec{C}$.

where $\hat{n}$ is the unit vector gives the direction of vector $\vec{C}$.

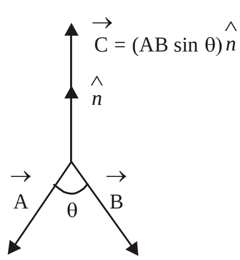
The unit vector $\hat{n}$ normal to the plane containing vectors $\vec{A}$ and $\vec{B}$ is given by

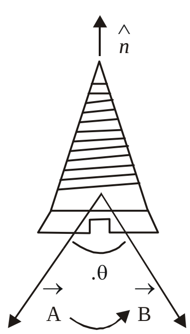


$$\hat{n} = \frac{\vec{A}\times\vec{B}}{AB\sin\theta} = \frac{\vec{A}\times\vec{B}}{\left|\vec{A}\times\vec{B}\right|}$$

Right Hand Rule

If a right handed screw is placed over the plane containing $\vec{A}$ and $\vec{B}$ as shown in the figure and is turned from $\vec{A}$ and $\vec{B}$ (anti-clockwise) through a small angle θ then the direction of advancement of the screw gives the direction of $\hat{n}$ or $\vec{A}\times\vec{B}$. i.e., upward perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.





Examples

(i) The cross product of anuglar velocity $(\vec{\omega})$ and the radius vector $(\vec{r})$ is equal to the velocity $(\vec{v})$ i.e., $\vec{v} = \vec{r}\times\vec{\omega}$

(ii) The cross prodcut of position vector $(\vec{r})$ and force $(\vec{F})$ is equal to the torque $(\vec{\tau})$ i.e.,

$$\vec{\tau} = \vec{r}\times\vec{F}.$$

(iii) Angular momentum $(\vec{L}) = \vec{r}\times\vec{P}$.

Properties of Vector Product or Cross Product

(i) Cross product of two vectors does not obey the commutative law.

i.e., $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

Here $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

(ii) Cross Product of two vectors is distributive over vector addition :

i.e. $\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

(iii) Cross Product of two Parallel vectors or equal vectors : In this case the angle between vectors will be zero degree.

$\therefore \quad \vec{A}\times\vec{B} = \left(AB\sin 0°\right)\hat{n} = 0 \qquad [\because \ \sin 0 = 0]$

The cross product of two equal vectors is given by

$$\vec{A}\times\vec{A} = \left(A\,A\sin 0°\right)\hat{n}$$

or $\vec{A}\times\vec{A} = 0$

Similarly,

$$\hat{i}\times\hat{i} = \left(1\times1\times\sin 0°\right)\hat{n} = 0$$
$$\hat{j}\times\hat{j} = \left(1\times1\times\sin 0°\right)\hat{n} = 0$$
$$\hat{k}\times\hat{k} = \left(1\times1\times\sin 0°\right)\hat{n} = 0$$

(iv) Cross Product of two pependicular vectors : In this case θ = 90°

$$A\times B = \left(AB\sin 90^{\circ}\right)\hat{n} = \left(AB\right)\hat{n}$$

(v) Cross Product between the pairs of Unlike Unit Vectors :

$$\hat{i}\times\hat{j} = 1\times1\times\sin 90°\,\hat{k} = \hat{k}$$

Similarly,

$$\hat{j}\times\hat{k} = \hat{i}$$
$$\hat{k}\times\hat{i} = \hat{j}$$

Now

$$\hat{j}\times\hat{i} = -\vec{k}$$
$$\hat{i}\times\hat{k} = -\hat{j}$$
$$\hat{k}\times\hat{j} = -\hat{i}$$

(vi) Cross Product of two vectors in terms of their Rectangular components

$$\vec{A} = x_1\hat{i}+y_1\hat{j}+z_1\hat{k} \text{ and } \vec{B} = x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$$

$$\therefore \quad \vec{A}\times\vec{B} = \left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)\times\left(x_2\hat{i}+y_2\hat{j}+z_2\hat{k}\right)$$

$$\vec{A}\times\vec{B} = x_1x_2\left(\hat{i}\times\hat{i}\right)+x_1y_2\left(\hat{i}\times\hat{j}\right)+x_1z_2\left(\hat{i}\times\hat{k}\right)$$
$$+y_1x_2\left(\hat{j}+\hat{i}\right)+y_1y_2\left(\hat{j}\times\hat{j}\right)+y_1z_2\left(\hat{j}\times\hat{k}\right)$$
$$+z_1x_2\left(\hat{k}\times\hat{i}\right)+z_1y_2\left(\hat{k}\times\hat{j}\right)+z_1z_2\left(\hat{k}\times\hat{k}\right)$$

$$\therefore \quad \vec{A}\times\vec{B} = +\left(y_1z_2-y_2z_1\right)\hat{i}+\left(x_2z_1-x_1z_2\right)j$$
$$+\left(x_1y_2-x_2y_1\right)\hat{k}$$

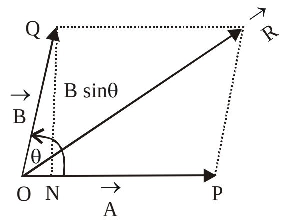
$\vec{A}\times\vec{B}$ can also be determined as follows:

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$= \hat{i}(y_1z_2 - y_2z_1) + \hat{j}$

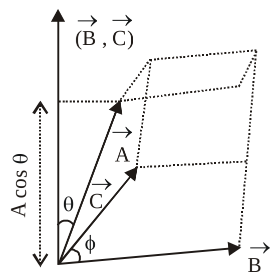
$(x_2z_1 - x_1z_2) + \hat{k}(x_1y_2 - x_2y_1)$

(vii) Magnitude of cross Product of two vectors $\vec{A}$ and $\vec{B}$ represents the area of the Parallelogram Suppose OPRQ be a parallelogram whose adjecent sides OP and OQ are represented both in magnitude and direction by two vectors $\vec{A}$ and $\vec{B}$.



$\therefore \quad |\vec{A} \times \vec{B}| = AB \sin\theta = A(B \sin\theta) = OP \times QN$ = Area of the parallelogram

Scalar Triple Product



$\vec{A} \cdot (\vec{B} \times \vec{C})$ is scalar and so is called scalar triple product. From the diagram,

$\vec{B} \times \vec{C} = BC \sin\phi$

Now, $\vec{A} \cdot (\vec{B} \times \vec{C}) = A \cos\theta\ (BC \sin\phi)$

= volume of parallelopiped

Regarding scalar triple product

It is worth noting that :

(i) It represents the volume of parallelopiped represented by edges $\vec{A}$, $\vec{B}$ and $\vec{C}$.

(ii) $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$ implies that vectors are coplanar.

(iii) In scalar tiple product dot and cross can be interchanged provided that their cyclic order is maintained.

(iv) Four points A, B, C and D are coplanar if $\overline{AB} \cdot (\overline{BC} \times \overline{CD}) = 0$

Multiple Choice Questions with One Correct Answer

1. A force $\hat{F} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ displaces an object from a point $P(1, 1, 1)$ to another point $Q(2, 0, 3)$. The work done by force is :

(a) 10 J (b) 12 J
(c) 13 J (d) none of these

2. Given $\vec{P} = P\cos\theta\,\hat{i} + P\sin\,\hat{j}.$ The vector $\vec{P}$ which is perpendicular to $\vec{Q}$ is given by :

(a) $Q\cos\theta\hat{i} - Q\sin\theta\hat{j}$ (b) $Q\cos\theta\hat{i} - Q\cos\theta\hat{j}$
(c) $Q\cos\theta\hat{i} + Q\sin\theta\hat{j}$ (d) $Q\cos\theta\hat{i} + Q\cos\theta\hat{j}$

3. A cyclist is moving on a circular path with constant speed. What is the change in its velocity after it has described an angle of 30º ?

(a) $v\sqrt{2}$ (b) $v(0.3\sqrt{3})$
(c) $v\sqrt{3}$ (d) none of these

4. The resultant of two vectors makes angle 30º and 60º with them and has magnitude of 40 unit. The magnitude of the two vectors are :

(a) 20 unit, 20 unit (b) 20 unit, 28 unit
(c) 20 unit, $20\sqrt{30}$ unit (d) 20 unit, 60 unit

5. A vector $\vec{P}$ makes an angle of 10º and $\vec{Q}$ makes an angle of 100º with x-axis. The maginitude of these vectors are 6 m and 8 m. the resultant of these vectors is :

(a) 10 m (b) 14 m
(c) 2 m (d) none of these

6. If $\vec{P} = \vec{Q} + \vec{R}$ and $\vec{Q} = \vec{R} + \vec{P}$ then the vector $\vec{R}$ is :

(a) $\vec{P} + \vec{Q}$ (b) $\vec{Q} - \vec{P}$
(c) $\vec{P} - \vec{Q}$ (d) null vector

7. What is the property of two vectors $\vec{P}$ and $\vec{Q}$ if $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$?

(a) P is null vector (b) Q is null vector
(c) P is proper vector (d) Q is proper vector

8. If $\vec{P} \times \vec{Q} = \vec{0}$ and $\vec{Q} \times \vec{R} = \vec{0},$ then the value of $\vec{P} \times \vec{R}$ is :

(a) zero vector (b) zero scalar
(c) unit vector (d) $PR\cos\theta$

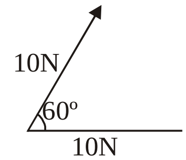
9. Resultant of which of the following may be equal to zero ?

(a) 10 N, 10 N, 10 N (b) 10 n, 10 N, 25 N
(c) 10 N, 10 N, 35 N (d) none of these

10. For two vectors $\vec{A}$ and $\vec{B}$, which of the following relations are not commutative ?

(a) $\vec{P} + \vec{Q}$ (b) $\vec{P} \times \vec{Q}$
(c) $\vec{P} \bullet \vec{Q}$ (d) none of these

11. Two vectors $\vec{A}$ and $\vec{B}$ are aciting as shown in figure. If $|\vec{A}| = |\vec{B}| = 10N$ hten the resultant is :



(a) $10\sqrt{2}N$ (b) 10 N

(c) $5\sqrt{3}N$ (d) none of these

12. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between A and B is :

(a) π (b) π/3

(c) π/2 (d) π/4

13. Let $\vec{A},\vec{B}$ and $\vec{C}$ be unit vectors. Suppose that $\vec{A}.\vec{B}=\vec{A}.\vec{C}=0$ and that the angle between $\vec{B}$ and $\vec{C}$ is $\frac{\pi}{6}$ then $\vec{A}$ equal to :

(a) zero (b) $(\vec{B}\times\vec{C}).\vec{B}$

(c) $2(\vec{B}\times\vec{C})$ (d) $(\vec{B}\times\vec{C})$

14. If $\vec{A}=2\hat{i}-3\hat{j}+4\hat{k},\ \vec{B}=\hat{i}+2\hat{j}$ and $\vec{C}=\hat{j}-\hat{k},$ then the value of $\vec{A}.(\vec{B}\times\vec{C})$ is :

(a) zero (b) 5

(c) 14 (d) 6

15. On one rainy day a car starts moving with a constant acceleration of 1.2 m/s^2. If a toy monkey is suspended from the ceilling of the car by a string, then at what angle the string is inclined with the vertical ?

(a) $\tan^{-1}(0.25)$ (b) $\tan^{-1}(0.63)$

(c) $\tan^{-1}(0.12)$ (d) $\tan^{-1}(\sqrt{3})$

16. A man walks 20 m at an angle of 60° north-east. How far towards east has he travelled ?

(a) 10 m (b) 20 m

(c) $20\sqrt{3}$ m (d) $\frac{10}{\sqrt{3}}$ m

17. A particle is moving along a circular path with a constant speed 30 m/s. What is change in velocity of a particle, when it describes an angle of 90° at the centre of the circle ?

(a) zero (b) $30\sqrt{2}$ m/s

(c) $60\sqrt{2}$ m/s (d) $\frac{30}{\sqrt{2}}$ m/s

18. If $\hat{A}$ is a unit vector, the value $\hat{A}.\frac{d\hat{A}}{dt}$ is :

(a) 0 (b) 1

(c) $\frac{\pi}{2}$ (d) π

19. If the three vectors are coplanar,

$$\vec{A}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{B}=x\hat{j}+3\hat{k}$$

and $$\vec{C}=7\hat{i}+3\hat{j}-11\hat{k}$$

then value of x is :

(a) 36/21 (b) – 51/32

(c) 51/32 (d) – 36/21

20. The three conterminous edges of a parallelopiped are

$$\vec{a}=2\hat{i}-6\hat{j}+3\hat{k},\vec{b}=5\hat{j}$$

and $\vec{c} = -2\hat{i} + \hat{k}$

The volume of parallelopiped is :

(a) 36 cubic unit (b) 45 cubic unit

(c) 40 cubic unit (d) 54 cubic unit

21. A force $\vec{F} = (2\hat{i} + 3\hat{j} - \hat{k})$ N is acting on abody at a position $\vec{r} = (6\hat{i} + 3\hat{j} - 2\hat{k})$. The torque about the origin is :

(a) $(3\hat{i} + 2\hat{j} + 12\hat{k})$ Nm (b) $(9\hat{i} + \hat{j} + 7\hat{k})$ Nm

(c) $(\hat{i} + 2\hat{j} + 12\hat{k})$ Nm (d) $(3\hat{i} + 12\hat{j} + \hat{k})$ Nm

22. Choose the correct option for $\vec{A} \times \vec{B} = \vec{C}$:

(i) $\vec{C}$ is perpendicular to $\vec{A}$

(ii) $\vec{C}$ is perpendicular to $\vec{B}$

(iii) $\vec{C}$ is perpendicular to $(\vec{A} + \vec{B})$

(iv) $\vec{C}$ is perpendicular to $(\vec{A} \times \vec{B})$

(a) Only (i) and (ii) are correct

(b) Only (ii) and (iv) are correct

(c) (i), (ii) and (iii) are correct

(d) All are correct

23. The value of $\hat{i} \times (\hat{i} \times \vec{a}) + \hat{j} \times (\hat{j} \times \vec{a}) + \hat{k}(\hat{k} \times \vec{a})$ is :

(a) $\vec{a}$ (b) $\vec{a} \times \hat{k}$

(c) $-2\vec{a}$ (d) $-\vec{a}$

24. If $\vec{c} = \vec{a} \times \vec{b}$, then :

(a) the direction of $\vec{c}$ changes, when the angle between $\vec{a} \times \vec{b}$ increases up to θ (θ < 180º)

(b) the direction of $\vec{c}$ changes, when the angle between $\vec{a}$ and $\vec{b}$ decreases up to θ (θ > 0º)

(c) the direction of $\vec{c}$ does not change, when the angle between $\vec{a}$ and $\vec{b}$ increases

(d) none of the above

25. The work done by a force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N, to displace a body from position *A* to position *B* is : [The position vector of *A* is $\vec{r_1} = (\hat{i} + 3\hat{j} + \hat{k})$ m and the position vector of *B* is $\vec{r_2} = (2\hat{i} + 2\hat{j} + 3\hat{k})$ m :

(a) 5 J (b) 3 J

(c) 2 J (d) 10 J

26. A force $\vec{F} = 2\hat{i} + 3\hat{j} + \hat{k}$ acts on a body. The work done by the force for a displacement of $-2\hat{i} + \hat{j} - \hat{k}$ is :

(a) 2 unit (b) 4 unit

(c) – 2 unit (d) – 4 unit

27. The velocity of a particle is $\vec{V} = 6\hat{i} + 2\hat{j} - 2\hat{k}$. THe component of the velocity of a particle parallel to vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ in vector form is :

(a) $6\hat{i} + 2\hat{j} + 2\hat{k}$ (b) $2\hat{i} + 2\hat{j} + 2\hat{k}$

(c) $\hat{i} + \hat{j} + \hat{k}$ (d) $6\hat{i} + 2\hat{j} - 2\hat{k}$

28. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the vector $\mathbf{Q}$ is eversed, then the resultant becomes $\vec{S}$, then choose the correct option.

(a) $R^2 + S^2 = 2 (P^2 - Q^2)$ (b) $R^2 + S^2 = 2 (P^2 + Q^2)$

(c) $R^2 + S^2 = (P^2 - Q^2)$ (d) $R^2 - S^2 = 2 (P^2 + Q^2)$

29. The angle between vector $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 3\ \hat{i} - 4\hat{j}$ is equal to :

(a) $\cos^{-1}\left(\frac{3}{15}\right)$ (b) $\cos^{-1}\left(\frac{1}{15}\right)$

(c) zero (d) $\cos^{-1}\frac{2}{15}$

30. A particle starts from rest at the origin with a constant acceleration $\vec{a} = 2\hat{i} + 8\hat{j} - 6\hat{k}$ ms^{-2}. Its position at t = 5*s* is :

(a) $(25\hat{i} + 100\hat{j} - 75\hat{k})$ m (b) $(25\hat{i} - 100\hat{j} - 75\hat{k})$ m

(c) $(100\hat{i} - 25\hat{j} + 75\hat{k})$ m (d) $(25\hat{i} - 100\hat{j} + 75\hat{k})$ m

31. Obtain the magnitude and direction cosines of vector $(\vec{A} - \vec{B})$, if $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{B} = 2\hat{i} + 2\hat{j} + 3\hat{k}$.

(a) $0, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$ (b) $0, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}$

(c) $0, 0, \frac{1}{\sqrt{5}}$ (d) none of these

32. The position vector of a moving particle at time *t* is $\vec{r} = 3\hat{i} + 4t^2\hat{j} - t^3\hat{k}$. Its displacement during the time interval *t* = 1 *s* to *t* = 3 *s* is :

(a) $\hat{j} - \hat{k}$ (b) $3\hat{i} + 4\hat{j} - \hat{k}$

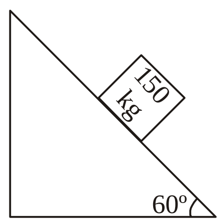
(c) $9\hat{i} + 36\hat{j} - 27\hat{k}$ (d) $32\hat{j} - 26\hat{k}$

33. A cat is situated at point *A* (0, 3, 4) and a rat is situated at point *B* (5, 3, –8). the cat is free to move but the rat is always at rest. The minimum distance travelled by cat to catch the rat is :

(a) 5 unit (b) 12 unit

(c) 13 unit (d) 17 unit

34. A block of 150 kg is placed on an inclined plane with an angle of 60º. The component of the weight parallel to the inclined plane is :



(a) 1300 N (b) 1400 N

(c) 1100 N (d) 1600 N

35. A man walks 4 km due west, 500 m due south finally 750 m in south west direction. The distance and magnitude of displacement travelled by the man are :

(a) 4646.01 m and 5250 m (b) 5250 m and 4646.01 m

(c) 4550.01 m and 2300 m (d) none of these

36. *ABCD* is a parallelogram, and $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of vertices *A, B, C* and *D* of a parallelogram. the correct option is :

(a) $\vec{c} + \vec{b} = \vec{d} - \vec{a}$ (b) $\vec{c} - \vec{b} = \vec{d} - \vec{a}$

(c) $\vec{b} - \vec{c} = \vec{d} - \vec{a}$ (d) none of these

37. Three forces are acted on a body. Their magnitudes are 3N, 4 N and 5 N. Then :

(a) the acceleration of body must be zero

(b) the acceleration of body may be zero

(c) the acceleration of the body must not be zero

(d) none of the above

38. How many minimum number of vectors of equal magnitude are required to produce zero resultant ?

(a) 2 (b) 3

(c) 4 (d) More than 4

39. The angle between $\vec{A}$ and hte resultant of $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ will be :

(a) 0º (b) $\tan^{-1}\left(\frac{A}{B}\right)$

(c) $\tan^{-1}\left(\frac{B}{A}\right)$ (d) $\tan^{-1}\left(\frac{A-B}{A+B}\right)$

40. If two forces of equal magnitude 4 units acting at a point and the angle between them is 120º then the magnitude and direction of the sum of the two vectors are :

(a) 4, $\theta = \tan^{-1}(1.73)$ (b) 4, $\theta = \tan^{-1}(0.73)$

(c) 2, $\theta = \tan^{-1}(1.73)$ (d) 6, $\theta = \tan^{-1}(0.73)$

41. Two forces of magnitudes 3N and 4N are acted on a body. The ratio of magnitude of minimum and maximum resultant forces on the body is :

(a) 3/4 (b) 4/3

(c) 1/7 (d) none of these

42. An insect moves on a circular path of radius 7m. The maximum magnitude of displacement of the insect is :

(a) 7 m (b) 14 π m

(c) 7 π m (d) 14 m

43. What is the maximum number of rectangular components into which a vector can be split in its own plane ?

(a) 2 (b) 3

(c) 4 (d) Infinite

44. Which of the following operations make no sense in case of scalars and vectors ?

(a) Multiplying any vector by a scalar

(b) Adding a component of vector to the same vector

(c) Multiplying any two scalars

(d) Adding a scalar to a vector of the same dimensions

45. Presure is :

(a) a scalar quantity (b) a vector quantity

(c) a tensor quantity (d) either scalar or vector

ANSWERS

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (a)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (b)	**28.** (b)	**29.** (d)	**30.** (a)
31. (a)	**32.** (d)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (b)	**37.** (a)	**38.** (a)	**39.** (a)	**40.** (a)
41. (c)	**42.** (d)	**43.** (a)	**44.** (d)	**45.** (a)					

HINTS & SOLUTIONS

3. $\Delta v = \vec{v}_2 - \vec{v}_1$

$$\Delta v = \sqrt{v^2 + v^2 + 2v^2 \cos(180^\circ - 30^\circ)}$$

$$= \sqrt{v^2 + v^2 + 2v^2 \times (-0.866)}$$

$$= \sqrt{2v^2 - 1.73v^2}$$

$$= \sqrt{0.27v^2} = \frac{v}{10} 3\sqrt{3}$$

4. $$\tan 60^\circ = \frac{Q \sin 90^\circ}{P + Q \cos 90^\circ} \Rightarrow \sqrt{3} = \frac{Q}{P}$$

$\therefore$ $Q = \sqrt{3}P$

$P^2 + Q^2 = 1600$

$\Rightarrow$ $P^2 + 3P^2 = 1600$

$\therefore$ $P = 20$ unit

Also $Q = \sqrt{3}P = 20\sqrt{3}$

8. $\vec{P} \times \vec{Q} = \vec{Q} \rightarrow \vec{P} \parallel \vec{Q}$

$\vec{Q} \times \vec{R} = \vec{0} \rightarrow \vec{Q} \parallel \vec{R}$

$\therefore$ $\vec{P} \times \vec{R} = \vec{0}$ then $\vec{P} \parallel \vec{R}$

9. The resultant of three vectors will be zero if and only if the sum of two smaller vectors is equal to or greater than third vector.

10. Vectors addition and scalar product of two vectors obey the commutative law.

i.e., $\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$

and $\vec{P}.\vec{Q} = \vec{P}.\,\vec{Q}$

But cross product of two vectors does not obey commutative law, that is

$$\vec{P} \times \vec{Q} = -\vec{Q} \times \vec{P}$$

or $\vec{P} \times \vec{Q} \neq \vec{Q} \times \vec{P}$

11. Here the angle θ between the two vectors is 120º and not 60º.

$\therefore$ $R = \sqrt{(10)^2 + (10)^2 + 2(10)(10)\cos 120^\circ}$

$$= \sqrt{100 + 100 - 100} = 10\ \text{N}$$

14. $\vec{A}.(\vec{B} \times \vec{C}) = [\vec{A}\vec{B}\vec{C}]$, volume of paralelopiped

$$= \begin{vmatrix} 2 & -3 & 7 \\ 1 & 2 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$= 2\,(-2-0) + 3\,(-1-0) + 7\,(1-0)$

$= -4 - 3 + 7 = 0$

18. We know $\hat{A}.\hat{A} = 1,$ where $\hat{A}$ is any unit vector.

$$\Rightarrow \quad \frac{d}{dt}(\hat{A}.\hat{A}) = 0$$

$$\Rightarrow \quad \hat{A}.\frac{d\hat{A}}{dt} + \hat{A}.\frac{d\hat{A}}{dt} = 0$$

$$\Rightarrow \quad 2\left(\hat{A}.\frac{d\hat{A}}{dt}\right) = 0$$

$$\Rightarrow \quad \hat{A}.\frac{d\hat{A}}{dt} = 0$$

20. Volume $V = \vec{a}.(\vec{b}\times\vec{c}) = \begin{vmatrix} 2 & -6 & 3 \\ 0 & 5 & 0 \\ -2 & 0 & 1 \end{vmatrix}$

$= 2\,(5) + 6\,(0) + 3\,(10)$

$= 10 + 30 = 40$ cubic unit

25. Work done $W = \vec{F}.\,\vec{s}$

s = displacement of the body $\vec{r}_2 - \vec{r}_1$

$= (2\hat{i} + 2\hat{j} + 3\hat{k}) - (\hat{i} + 3\hat{j} + \hat{k})$

$= (\hat{i} - \hat{j} + 2\hat{k})$ m

$W = (\hat{i} + 2\hat{j} + 3\hat{k}).(\hat{i} - \hat{j} + 2\hat{k})$

$= 1 - 2 + 6 = 5$ J

29. $\vec{a}\bullet\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\cos\theta = \frac{\vec{a}\bullet\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(2\hat{i} + \hat{j} - 2\hat{k})\bullet(3\hat{i} - 4\hat{j})}{\sqrt{2^2 + 1^2 + (-2)^2}\sqrt{(3)^2 + (-4)^2}}$$

$$= \frac{6-4}{3\times 15} = \frac{2}{15} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{15}\right)$$

31. $(\vec{A} - \vec{B}) = 2\hat{i} + 3\hat{j} + \hat{k} - 2\hat{i} - 2\hat{j} - 3\hat{k}$

$= \hat{j} + 2\hat{k}$

$|\vec{A} - \vec{B}| = \sqrt{1+4} = \sqrt{5}$

Direction cosines $= \frac{0}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$ *i.e.*, $0, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$

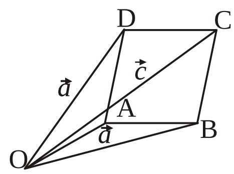
36. As shown in figure, according to condition given in question

$\overrightarrow{BC} = \vec{c} - \vec{b}$

$\overrightarrow{AD} = \vec{d} - \vec{a}$

but $\overrightarrow{BC} = \overrightarrow{AD}$

$\therefore$ $\vec{c} - \vec{b} = \vec{d} - \vec{a}$



38. Two vectors of equal magnitude and directed in opposite direction give zero resultant.

40. $|\vec{R}| = \vec{a} + \vec{b}| = \sqrt{4^2 + 4^2 + 2(4)(4)\cos 120^o}$

$|\vec{R}| = 4$

Let θ = angle made by $\vec{a} + \vec{b}$ with *x*-axis

$$\theta = \tan^{-1}\left(\frac{4\sin(120^o)}{4 + 4\cos(120^o)}\right) = \tan^{-1}\frac{(3.464)}{2}$$

$$= \tan^{-1}(1.73)$$

42. time$= \frac{\text{distance}}{\text{speed}} = \frac{\pi r}{10} = \frac{22}{7} \times \frac{7}{10} = 2.2s$

43. The number of rectangular components into which a vector can be splitted in its own plane is two because a plane is two-dimensional.

44. A scalar cannot be added to a vector.